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Haskell at PIK

Haskell in Leipzig

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## Formalising “Vulnerability”

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- System:

$f :: X \rightarrow F X$

We consider only the case where  $F$  is a *monad*

$\text{data (Monad m) } \Rightarrow \text{ Sys m a = Sys (a } \rightarrow \text{ m a)}$

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- A system can be *applied*:

$\text{apply} :: (\text{Monad } m) \Rightarrow \text{Sys } m \ a \ \rightarrow \ m \ a \ \rightarrow \ m \ a$

$\text{apply } \text{sys } x = x \gg= \text{sys}$

and *iterated*:

$\text{iter} :: (\text{Monad } m) \Rightarrow \text{Sys } m \ a \ \rightarrow \ m \ a \ \rightarrow \ \text{Int} \ \rightarrow \ m \ a$

$\text{iter } \text{sys } x \ n = \text{take } n \ (\text{iterate } (\gg= \text{sys}) \ x)$

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- Examples:

m = Id Deterministic

m = [] Non-deterministic

m = SimpleProb Stochastic (M. Erwig)

Most important: *Combinations* of these.

## Formalising “Vulnerability”

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- *Deterioration*

worse\_than :: (Monad m) => m a -> m a -> Bool

a partial strict order

(apply sys x) ‘worse\_than‘ x

expresses a general idea of *deterioration* of the state.

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## Formalising “Vulnerability”

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- *Vulnerability*

Deterioration in the context of a *socio-ecological* system:

$$\text{sys} :: (X, Y) \rightarrow m (X, Y)$$

or

$$\text{soc\_sys} :: (X, Y) \rightarrow m X$$
$$\text{eco\_sys} :: (X, Y) \rightarrow m Y$$
$$\text{sys} = \text{tau} . \text{pair}(\text{soc\_sys}, \text{eco\_sys})$$

## Formalising “Vulnerability”

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- Problems:
  - Given a non-deterministic `soc_sys` and a stochastic `eco_sys`, how can they be combined?
  - Check that *worse\_than* is a strict order
  - Extend strict orders from  $a \rightarrow a$  to  $m\ a \rightarrow m\ a$
  - **Etc.**

## The S model for parallel computations

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- Formalisation of BSP
- Main elements:
  - Distributed data newtype  $D\ a = D\ (\text{Proc} \rightarrow a)$  e.g.  $\text{pid} = D\ \text{id}$
  - Monad definition used to describe local computations



## The S model for parallel computations

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- Main elements (cont.):
  - Communication primitive:  $\text{exch} :: D [(a, \text{Proc})] \rightarrow D [(a, \text{Proc})]$
  - *Reduction* of a constant value:  
 $\text{val } dx = dx \ 0$ , if  $dx$  is constant  
undefined otherwise.

# The S model for parallel computations

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- Usage:
  - Formulate problems
  - Implement and test proposed solution
  - Serve as documentation of C++ implementation